Analytic Problem Solving Methodology

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Abstract

This paper examines the largely ignored process of analytic problem solving. By applying learning theory to different problem-solving situations, we determine what factors contribute to a positive learning environment. An ideal environment includes facilities to observe the problem states simply and easily, and guidance in abstracting the observations into symbolic expressions.

Introduction

Engineering education has traditionally focused on analytic problem-solving skills. Within the last few decades educators have also begun to explore the realm of open-ended design exercises, with a focus on design methodology. Modern engineering curricula typically begin with analytical 'basics' and then jump abruptly into design project courses. Very recently educators and researchers have begun to formulate ways to bridge the gap between the analytical and design-oriented worlds of engineering curricula.

The work on design process and methodology has advanced considerably. Analytical process, however, has remained largely unexplored. In analytic courses, lecture and problem set formats predominate. Little or no attention is paid to the process or method of solving the problem; the focus is on determining the correct answer. This is an unrealistic and problematic standard to set because there is rarely a single correct answer in professional engineering practice. This focus deepens the dichotomy between analytical and design skills. The neglect of attention to analytical process also ignores important cues for gauging the efficacy of the curriculum. Attention to these cues could reveal problematic concepts in the curriculum and point to ways to overcome barriers to learning the material.

This paper is an initial investigation into the realm of analytical process. The project was motivated by a desire to understand how students use multiple resources in their solution methodology. The material is based on a series of observations of students attempting to solve a problem with a variety of analytical tools at their disposal. Their solution process is examined with regard to principles of learning theory which serves to illuminate and explicate differences in the educational value of the exercise for the participants.

Scenario

Four pairs of students were videotaped working on an assignment for an upper level (junior and senior), undergraduate, mechanical design course. This paper will focus on three of the pairs. [The audio portion of the fourth videotape was lost, due to technical difficulties.]

The assignment was well-defined. It required the students to design a four-bar toggle clamp mechanism with specified force-magnification and dimensional criteria. A range of solutions would meet the criteria and be considered correct solutions. The assignment sheet contained a diagram of a mechanism similar to those which would be considered correct solutions. The students needed to determine the relative lengths and positions of the four links in the mechanism, such that the force and length criteria were met.

All of the participants had access to a multi-page handout which had accompanied a lecture on analyzing force-magnification of four-bar mechanisms. The handout outlined a procedure for performing such analyses using principles of energy conservation, geometry, and algebra. The technique is called 'instant-center analysis' and is based on a concept called 'virtual work.' The handout also contained an example analysis for a four-bar mechanism of a similar type to a toggle clamp.

The participants had also taken part in an hour-long tutorial on the use of a mechanical simulation software tool called Working Model™. The tutorial introduced the basic techniques required to build a model and run a simulation, with a special focus on four-bar mechanisms.

All six of the participants were male, and roughly the same age. According to questionnaires completed prior to taping, they were of similar social backgrounds, and had similar engineering experience and mechanical aptitude. They were all novices with respect to instant-center analysis, and the design of four-bar mechanisms.

All three pairs had access to a work table, paper, pencils, rulers, calculators, the assignment sheet and the handout mentioned above. The 'paper' pair, JS and DB, had only these resources available. The 'computer' pair, EM and CD, had access to all of the same tools, plus a powerful personal computer with Working Model software running on it. The 'Lego' pair, MM and BM, had access to all of the previous tools, in addition to a Technics Lego™ building set, provided at MM's request.

Observations

The participants were taped for one hour while they worked on the problem. This is a brief synopsis of the activity on the tapes for each of the three pairs.

Paper Pair

JS entered the room a few minutes after DB. JS began to read the assignment while DB operated the computer. About five minutes later, DB stated, "They must not want us to use Working Model, 'cause it's not here." DB moved back to the work table. JS asked DB to describe his 'approach' to the problem. JS did not wait for an answer, and began to restate the criteria in the assignment.

After some discussion, the participants began to apply the procedure laid out in the handout to their problem. They ran into several snags in the course of the procedure, regarding issues of force component vectors and nomenclature. They applied various criteria, performed some calculations and determined that the length of one of the links in the mechanism should be -32 centimeters. The negative sign indicated that this was an erroneous result. They spent the remainder of the hour attempting to find the source of the error in their calculations, but did not succeed.

After the taping session, the students successfully completed the analytical procedure, and turned in a paper detailing their analysis and the resulting mechanism. They included a cardboard model of the clamp that they constructed.

Computer Pair

As EM read through the handout on virtual work, CD moved to the computer. EM agreed that "Working Model might be a better way to go" and joined CD at the computer. EM began to enumerate the specifications for the toggle clamp, using hand gestures and the diagram in the handout to illustrate. EM wanted to create a model that had the appropriate behavior, but he believed that "there's a bunch of numerical stuff that Working Model can't do." CD began building a clamp model with the software. The resulting model had two toggle conditions, which the pair found undesirable. They reworked the model twice, attempting to eliminate the second toggle. They discovered a consistent relationship between the lengths of the links. They believed that this concept would allow them to eliminate the unwanted behavior.

They started over with a blank screen, attempting to apply the new concept to a new model. Their first attempt was abandoned, but their second restart resulted in a mechanism that had only one toggle condition and appeared to meet the problem specifications. They measured the link lengths and proposed scaling the lengths to meet the size specifications.

After the taping session, the students scaled down the software model. They used the instant-center analysis technique on the model to show that it provided the specified force magnification. They turned in a cardboard model of the clamp mechanism.

Lego Pair

Prior to the beginning of taping, a Lego set was provided to the pair at MM's request. Both students began to build models of a toggle clamp with the Legos, based on the diagram in the assignment sheet. MM stated that he wanted to "get a model that works pretty well." Both students constructed models of the appropriate class. They referred to the criteria in the assignment several times, and revised their models. After about fifteen minutes, they compared the two Lego models and chose BM's, because MM's required a starting position that he deemed undesirable.

BM measured the links in his model and MM constructed a scaled model with the Working Model software. MM worked with the model for about 30 minutes with intermittent input from BM. MM altered the model as he considered the angular and dimensional criteria in the problem statement. Meanwhile, BM read through the handout and decided that the "instant-center stuff is ugly." MM proposed that the force analysis could be done with the simulation software.

After the taping session, the students successfully analyzed the force ratios with the simulation software. They turned in computer print-outs from the simulation software which depicted the forces at various states of the linkage. They also turned in BM's Lego model of the clamp.

Analysis

We will begin the analysis section with some background on situated learning theory. This theory is by no means the only one which describes the manner in which people learn, but it is a complete, functional and widely-accepted theory. For a working definition of learning, we turn to Greeno, Moore and Smith. They state that, "*knowing* is ability to interact with things and other people in a situation, and *learning* is improvement in that ability" [1]. Learning depends on, and is influenced by, the situation in which the learning takes place. The situation is created by the properties of the learning environment and the characteristics of the learners, or *agents*. An *affordance* is "a resource in the environment that supports an interactive activity by an agent" [2]. The "characteristics of the agents that enable them to engage in activities"[1] are called *abilities*.

In these observations, the situations differed primarily in the realm of affordances. Each student certainly had different abilities, but, because of the similar backgrounds and levels of training and expertise, the differences were relatively small. There were four different affordances available in the experiment which influenced the solution methods and strategies, or *schema*, that were utilized.

The **first** affordance included the **work table, paper, pencils and the handout** describing instant-center analysis. This set of items afforded the participants the ability to follow the example analysis and apply the technique to the toggle clamp problem

The **second** affordance was the **Working Model** mechanical simulation software. The software afforded the ability to create two-dimensional models of mechanisms that functioned like real mechanisms. It also included the ability to make changes to the dimensional aspects of the model and observe the effects immediately.

The **third** affordance was the **analytical capability of the Working Model software**. This capability was briefly introduced to the participants during the tutorial. The tutorial focused on the second affordance, however, and the analytical capability was essentially an aside.

The **fourth** affordance was the **Lego Technics building set**. The Legos afforded the ability to create approximate physical models of mechanisms. This is a fundamentally different activity than creating software models. The tactile aspect of this activity, the ability to touch the mechanism and feel relative forces, distinguishes it from the software environment. The software environment, however, provided the ability to make minute and/or rapid changes in the model.

The solution schemas that the three pairs chose depended greatly on the affordances available to them. The paper pair was limited to an example-following schema, since they only had the first affordance available to them. It was, however, a familiar schema; the use of example problems to teach new analytical techniques is quite common in engineering education.

The computer pair used the software to develop a model that behaved correctly according to the qualitative specifications, then followed the analytical example to determine the quantitative behavior. Their solution utilized the successive-approximation affordance of the software, and the analytical affordance of the example.

The Lego pair had access to all four affordances, but only used three of them in their solution. They developed an initial physical model with the Legos, then performed finer successive approximations with the software. They then used the analytical capabilities of the software to determine the quantitative results. They did not employ the first affordance at all.

In the process of solving the problem, each pair of participants completed the assignment satisfactorily and created mechanisms that met the stated criteria. So, by our working definition, they all learned about designing mechanisms because they interacted ‘successfully’ in their situations. Each situation was distinctly different, however, primarily because of the different affordances available. But how can we determine which situation enabled the most effective learning environment and brought about the highest level of understanding?

The concept of *representations* allows us to differentiate the effectiveness of the learning environments and the learning experience of each pair. Representations are expressions, usually symbolic in nature, of the actual or potential states of a situation. There are two primary means of constructing representations: physically and mentally. *Physical representations* “include physical constructions such as diagrams, graphs, pictures, and models with properties that are interpreted as corresponding to properties of situations” [1]. *Mental representations* include mental constructions such as symbolic expressions and mental models [1]. *Mental models* feature cognitive objects which correspond to physical objects, situations, or relationships and that can be used to simulate actions or events in other situations [1]. When a student is able to form a complete and functional mental model of a situation, we say he *understands* that subject. Of course we cannot know for certain what mental models of mechanisms the participants form as a result of this exercise, but we can determine what mental representations are possible based on the physical representations available to them.

The paper pair has some diagrams and several equations to serve as their available representations. Four-bar mechanisms are simple to construct, but they can involve rather complex motions. These motions are difficult to convey in a simple, static diagram. The diagrams available to the paper pair were probably not sufficient to allow the participants to form a complete mental representation of the situation. The paper pair was focused primarily on a symbolic representation of the problem, i.e., the equations involved in the instant-center analysis. Such a symbolic model, even if complete and functional, rarely leads to a complete mental model.

There is evidence of the incompleteness of this pair's mental model when they completed a series of computations and concluded that the length of one of the links should be -32 cm. The negative sign was an immediate indicator that the result was invalid, but the participants did not know where the error might have occurred. This is a classic example of the dissociation of a symbolic expression from the situation it describes [3]. The symbolic representation does not have a reliable link to the real-world mechanism in the mental model. If the participants had such a link in the mental model, they would be able to determine which procedure in the symbolic manipulation process was incompatible with the situation and caused the error. This type of error is not at all uncommon in institutional engineering education.

The Lego pair should have had an excellent mental representation of the situation. Both the Lego model and the software model aided in the construction of an accurate mental model of the situation. The interactivity of both models give it obvious advantages over a static diagram. The ability to experiment in the software environment, quickly and easily, allowed the participants to discover patterns in the behavior of the mechanisms. This would eventually enable the participants to make accurate qualitative predictions about the behavior in new, or altered models.

According to Nathan et al., there are two means of acquiring the conceptual entities required to construct a solution-enabling mental model: *observation* and *computation* [3]. The observational technique corresponds roughly to the observation of physical representations, while the computational technique corresponds to the application of symbolic representations. The Lego pair used observational techniques extensively in their solution, but severely neglected computation. They did obtain some quantitative data by using the analytic features of the software, but this did not involve computation or symbolic manipulation. While the Lego pair had an excellent situational understanding of the problem, they lacked a symbolic representation to complete the mental model.

The manner in which they determined the force ratio produced by the mechanism provides evidence of the incompleteness of their mental model. The software will not display forces directly, so the participants added springs to the model, and were able to determine the forces by measuring the displacements of the springs and multiplying by a known spring constant. The software will display velocity directly, however. The virtual work concept that is central to instant-center analysis states that the product of force and velocity at the input is equal to the product at the output. If the participants had grasped this symbolic concept, they would have been able to obtain a ratio of velocities directly from the software, and invert it to determine the force ratio.

The computer group should have formed the most complete mental model of mechanisms. This is because they employed both the observational and computational means of acquiring conceptual entities. Like the Lego pair, they began with the highly-effective, graphic, interactive software environment to develop an intuition about the mechanism. They then moved to the symbolic formalisms presented in the instant-center analysis. They should, therefore, have developed a complete and effective situational understanding, as well as an abstract symbolic understanding that, if linked properly, form a complete mental model of the problem. This pair should have had a good ‘feel’ for the problem, which would allow them to methodically track-down errors such as the one that caused the paper pair difficulties. A rich situational experience, when linked to an accurate abstract understanding, leads to an excellent opportunity for students to *transfer* their knowledge into new domains.

Since the pair did not perform the analysis on tape, we cannot cite evidence of the situational and symbolic representations linking into a complete mental model. We can, however, cite an example of an abstractable insight that the pair makes in their observations. When the pair was trying to remove the second toggle condition, they discovered a relationship between the link lengths which insured that one of the links would be able to perform a full rotation. This is called Grashof's criterion, and can be expressed in the form of an algebraic equation.

Conclusions

The primary conclusion we draw from these observations is that both the quantity and type of resources available to students determine the effectiveness of the learning environment. In the ideal environment, a rich situational experience builds a student’s intuition about the domain. This experience is followed by the development of abstractable symbolic relationships. When these two experiences are properly linked, the student develops a complete mental model which allows him to transfer his knowledge to other domains.

Engineering education has traditionally been quite constrained, focusing on procedural analysis and abstract reasoning. The lecture and problem-set format of instruction remains the predominate approach in most engineering institutions today. As demonstrated above, this format neglects the situational aspect of mental model construction.

Multimedia systems offer a means to round out the traditional techniques. Computer tools can provide excellent opportunities to add graphical, interactive exercises to engineering curriculum in a convenient, inexpensive manner. This can help students build situational representations which will enhance their intuitive understanding of engineering concepts. Many traditional instructors, however, have legitimate fears about students learning computer-based tools without understanding the underlying theory. An engineer who does not understand this background will not be able to determine whether the results are inaccurate or skewed because of simplifications and approximations inherent in the simulation.

As demonstrated in the analysis, there is a happy medium between the approaches. A mixture of abstract theory and hands-on experience provides a better learning environment than either approach does alone. An educational program which promotes construction of complete mental models is an achievable goal. By incorporating hands-on experiences, and introducing simulation tools to the traditional theoretical instructional environment, educators can enhance the learning environment, and increase understanding.

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